1) $M = \pm 9 \dfrac{\vec{E} \times \vec{F}}{|\vec{E} \times \vec{F}|}$

cross product because $\vec{M} \perp (\vec{E} \,\&\, \vec{F})$

$$\vec{E} \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & -2 \\ 0 & 3 & -6 \end{vmatrix}$$

$$= (-12+6)\hat{a}_x + 6\hat{a}_y + 3a_z = -6\hat{a}_x + 6\hat{a}_y + 3\hat{a}_z$$

Now to find the vector with magnitude (9) and direction of ($\vec{E} \times \vec{F}$) so we find the unit vector of $\vec{E} \times \vec{F}$

$$\hat{a}_{\vec{E}\times\vec{F}} = \pm \frac{\vec{E} \times \vec{F}}{|\vec{E} \times \vec{F}|}$$

$$\vec{M} = \pm 9 \left(\frac{-6\hat{a}_x + 6\hat{a}_y + 3\hat{a}_z}{\sqrt{36+36+9}} \right) = \pm 9 \left(\frac{-6\hat{a}_x + 6\hat{a}_y + 3a_z}{\sqrt{81}} \right)$$

$$= \pm (-6\hat{a}_x + 6\hat{a}_y + 3a_z)$$

2) Remember :- Solenoidal $\Rightarrow$ Divergence of vector $= 0$

conservative or irrotational $\Rightarrow$ curl of vector $= 0$

a) $\vec{A} = x^2 \hat{a}_x - 2xy\, a_y$

$\nabla \cdot \vec{A} = 2x - 2x = 0$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & -2xy & 0 \end{vmatrix} = (-2y)\hat{a}_z \neq 0$$

$\vec{A}$ is solenoidal but not conservative.

②

$\vec{B} = x^2 \hat{a}_x - y^2 \hat{a}_y + 2\hat{a}_z$

$\nabla \cdot \vec{B} = 2x - 2y \neq 0$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & -y^2 & 2 \end{vmatrix} = 0$$

$\vec{B}$ is conservative
or irrotational
but not solenoidal.

$\vec{C} = \frac{\sin(\phi)}{\rho^2} \hat{a}_\rho + \frac{\cos(\phi)}{\rho^2} a_\phi$

$$\nabla \cdot \vec{C} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho\left(\frac{\sin(\phi)}{\rho^2}\right)\right) + \frac{1}{\rho}\frac{\partial}{\partial \phi}\left(\frac{\cos(\phi)}{\rho^2}\right)$$

$$= \frac{-\sin(\phi)}{\rho^3} - \frac{\sin(\phi)}{\rho^3} = \frac{-2\sin(\phi)}{\rho^3} \neq 0$$

$$\nabla \times \vec{C} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{\sin(\phi)}{\rho^2} & \frac{\cancel{\rho}\cos(\phi)}{\rho^{\cancel{2}}} & 0 \end{vmatrix} = \left(\frac{-\cos(\phi)}{\rho^3} - \frac{\cos(\phi)}{\rho^3}\right)\hat{a}_z$$

$$= -\frac{2\cos(\phi)}{\rho^3}\hat{a}_z \neq 0$$

so $\vec{C}$ is neithe solenoidal nor conservative.

$D = re^{-r}\hat{a}_r$

$$\nabla \cdot \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 e^{-r}\right) = \frac{1}{r^3}\left(-r^3 e^{-r} + 3r^{\cancel{2}} e^{-r}\right) = \frac{1}{r}\left(-re^{-r} + 3e^{-r}\right) \neq 0$$

$$\nabla \times \vec{D} = \frac{1}{r^2 \sin(\theta)} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin(\theta)\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ re^{-r} & 0 & 0 \end{vmatrix} = 0$$

D is conseravative
but not solenoidal.

③ $\vec{A} = r\cos(\theta)\,\hat{a}_r + r\sin(\theta)\,\hat{a}_\theta + r\sin(\theta)\cos(\phi)\,\hat{a}_\phi$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3\cos(\theta)\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}\left(r\sin^2(\theta)\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\phi}\left(r\sin(\theta)\cos(\phi)\right)$$

$$= 3\cos(\theta) + \frac{1}{\cancel{r\sin(\theta)}}\left(2\cancel{r\sin(\theta)}\cos(\theta)\right) - \frac{1}{\cancel{r\sin(\theta)}}\cancel{r\sin(\theta)}\sin(\phi)$$

$$= 3\cos(\theta) + 2\cos(\theta) - \sin(\phi) = 5\cos(\theta) - \sin(\phi)$$

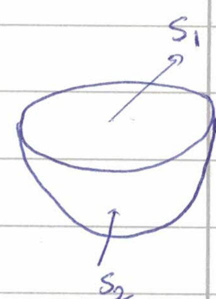
$$\iiint_V \nabla\cdot\vec{A}\,dv = \int_0^{2\pi}\int_{\pi/2}^{\pi}\int_0^{R}\left(5\cos(\theta) - \sin(\phi)\right)r^2\sin(\theta)\,dr\,d\theta\,d\phi$$

$$= \int_0^R r^2\,dr \int_{\pi/2}^{\pi}\left(\int_0^{2\pi}\left(5\cos(\theta) - \sin\phi\right)d\phi\right)\sin(\theta)\,d\theta$$

$$= \frac{R^3}{3}\left[\int_{\pi/2}^{\pi}\left(5\cos(\theta)(\phi) + \cos(\phi)\right)\Big|_0^{2\pi}\sin(\theta)\,d\theta\right]$$

$$= \frac{R^3}{3}\left[\int_{\pi/2}^{\pi}10\pi\cos(\theta)\,\sin(\theta)\,d\theta\right]$$

$$= \frac{10\pi R^3}{3}\left(\frac{\sin^2(\theta)}{2}\right)\Bigg|_{\pi/2}^{\pi} = -\frac{5\pi R^3}{3}$$



$\Psi_{total} = \Psi_1 + \Psi_2$

for $S_1 \Rightarrow \theta = \frac{\pi}{2}$, $\quad \vec{ds} = -r\sin(\theta)\,dr\,d\phi\,\hat{a}_\theta$

$$\Psi_1 = -\int_0^{2\pi}\int_0^{R} r^2\sin^2\theta\,dr\,d\phi = -\int_0^{2\pi}d\phi\int_0^R r^2\,dr = -\frac{2\pi R^3}{3}$$

Ψ_2 : for S_2 $r=R$, $\theta : \frac{\pi}{2} \to \pi$, $\phi : 0 \to 2\pi$

$$\vec{ds} = r^2 \sin(\theta)\, d\theta\, d\phi\ \hat{a}_r$$

$$\Psi_2 = \int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi} r^3 \cos(\theta)\sin(\theta)\, d\theta\, d\phi$$

$$\Psi_2 = R^3 \int_{\frac{\pi}{2}}^{\pi} \cos(\theta)\sin(\theta)\, d\theta \int_0^{2\pi} d\phi = R^3 \left(\frac{\sin^2(\theta)}{2}\right)_{\frac{\pi}{2}}^{\pi} \left(\phi\right)_0^{2\pi}$$

$$= -\frac{R^3}{\cancel{2}}(\cancel{2}\pi) = -\pi R^3$$

$$\Psi_{total} = \Psi_1 + \Psi_2 = -\frac{2\pi R^3}{3} - \pi R^3 = -\frac{5\pi R^3}{3}$$

$$\oint\!\!\oint \vec{A} \cdot \vec{ds} = \iiint_V \nabla \cdot A \, dv = -\frac{5\pi R^3}{3} \quad \#$$

5 $\vec{A} = xy\,\hat{a}_x + 2yz\,\hat{a}_y + 3xz\,\hat{a}_z$ $\qquad \underline{\underline{l=2}}$

$$\nabla \cdot \vec{A} = y + 2z + 3x$$

$$\int_V (\nabla \cdot A)\, dv = \int_0^2\int_0^2\int_0^2 (y + 2z + 3x)\, dx\, dy\, dz$$

$\frac{3x^2}{2}\Big|_0^2$

$$= \int_0^2\int_0^2 (2y + 4z + 6)\, dy\, dz$$

$\frac{2y^2}{2}\Big|_0^2 = 4 + 12$

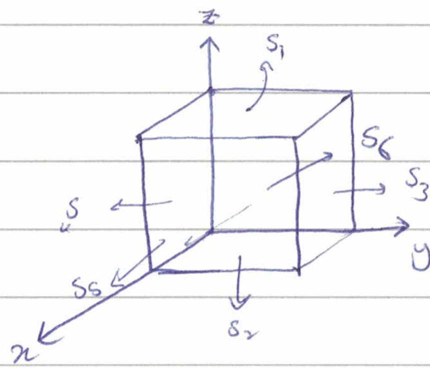
$$= \int_0^2 (16 + 8z)\, dz = \left|16z + 4z^2\right|_0^2 = 32 + 16 = 48$$

$$\psi_1 = \iint_{S_1} \vec{A}\cdot\vec{ds_1} = \int_0^2\int_0^2 3xz \, dx\, dy$$

$$= 3(z) \int_0^2 x \int_0^2 dy$$

$$= (6) \left.\frac{x^2}{2}\right|_0^2 \left. y \right|_0^2$$

$$= 6 \frac{(4)}{\cancel{2}} (2) = \underline{\underline{24}}$$



$$\psi_2 = \iint_{S_2} \vec{A}\cdot\vec{ds_2} = -\underline{\underline{3z}} \int_0^2\int_0^2 x\, dx\, dy = \underline{\underline{0}}$$

$$\boxed{z = 0}$$

$$\psi_3 = \iint_{S_3} \vec{A}\cdot\vec{ds_3} = 2y \int_0^2\int_0^2 z\, dx\, dz = 4(2)(2) = 16$$

$$\psi_3 = \iint_{S_4} \vec{A}\cdot\vec{d_{S4}} = -2\underline{\underline{y}} \int_0^2\int_0^2 z\, dx\, dz = 0$$

$$\psi_5 = \iint_{S_5} \vec{A}\cdot\vec{ds_5} = x \int_0^2\int_0^2 y\, dy\, dz = (2)(2)(2) = \underline{\underline{8}}$$

$$\psi_6 = \iint_{S_6} \vec{A}\cdot\vec{ds_6} = \underline{\underline{-x}} \int_0^2\int_0^2 y\, dy\, dz = 0$$

$$\psi_{total} = 24 + 0 + 16 + 0 + 8 + 0 = \underline{\underline{48}}$$

$$\oiint_S \vec{A}\cdot\vec{ds} = \iiint_v (\nabla\cdot A)\, dv = 48 \,\#$$

4

position vector $\vec{r} = \rho\hat{\rho} + z\hat{z}$

In this case position vector is equal to displacement vector as the reference point is the orign.

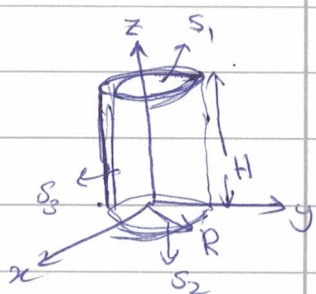
$$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

we know that $x\hat{a}_x + y\hat{a}_y = \rho\hat{a}_\rho$

$$\vec{r} = \rho\hat{a}_\rho + z\hat{a}_z$$

$$\nabla\cdot\vec{r} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2) + \frac{\partial}{\partial z}(z) = 2+1 = 3$$

$$\iiint_V \nabla\cdot\vec{r}\,dv = \int_0^H\int_0^{2\pi}\int_0^R 3\rho\,d\rho\,d\phi\,dz = 3\frac{R^2}{2}(2\pi)(H)$$
$$= \underline{3\pi R^2 H}$$



$S_1 \Rightarrow z = H,\ \rho: 0 \to R,\ \phi: 0 \to 2\pi$

$$ds_1 = \rho\,d\rho\,d\phi\,\hat{a}_z$$

$$\Psi_1 = \int_0^{2\pi}\int_0^R z\rho\,d\rho\,d\phi = H\frac{R^2}{2}(2\pi) = \pi R^2 H$$

$S_2:\ z = 0 \Rightarrow \Psi_2 = 0$ ~~, ds~~

$S_3:-\ \rho = R,\ z: 0 \to H,\ \phi: 0 \to 2\pi$

$$ds_3 = \rho\,d\phi\,dz\,\hat{a}_\rho$$

$$\Psi_3 = \rho^2\int_0^H\int_0^{2\pi} d\phi\,dz = R^2(2\pi)(H) = 2\pi R^2 H$$

$$\Psi_{total} = \pi R^2 H + 2\pi R^2 H = 3\pi R^2 H$$

$$\oint_S \vec{A}\cdot\vec{ds} = \iiint_V (\nabla\cdot\vec{r})\,dv = 3\pi R^2 H \quad \#$$

4 Position vector in cylindrical coordinates 7

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$A_\rho = x\cos(\phi) + y\sin(\phi)$$
$$A_\phi = -x\sin(\phi) + y\cos(\phi)$$
$$A_z = z$$

$$x = \rho\cos(\phi), \quad y = \rho\sin(\phi)$$

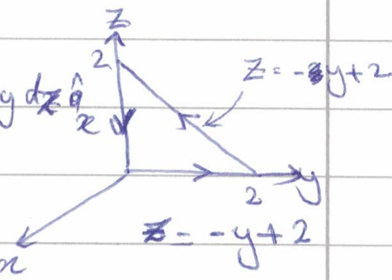
$$A_\rho = \rho\cos^2(\phi) + \rho\sin^2(\phi) = \rho$$

$$A_\phi = -\rho\sin(\phi)\cos(\phi) + \rho\sin(\phi)\cos(\phi) = 0$$
$$A_z = z$$

$$\boxed{\vec{A} = \rho\,\hat{a}_\rho + z\,\hat{a}_z} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

6 $\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = 2y\,\hat{a}_x - 3z\,\hat{a}_y + x\,\hat{a}_z$

$\int (\nabla \times \vec{A})\,\vec{ds} = \int (-2y\,\hat{a}_x - 3z\,\hat{a}_y - x\,\hat{a}_z)\, dy\, dz\, \hat{a}_x$



$= \int_s -2y\, dy\, d_z = \int_0^2 \int_0^{-y+2} -2y\, dz\, dy$

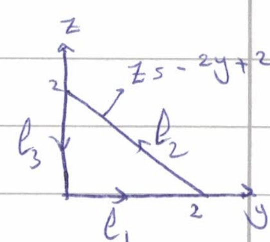
$= \int_0^2 -2yz\Big|_0^{-y+2} dy$

$= \int_0^2 \left(-2y(-y+2)\right) dy$

$= \int_0^2 (2y^2 - 4y)\, dy = \left(\frac{2y^3}{3} - 2y^2\right)\Big|_0^2$

$= \frac{2(8)}{3} - 2(2)^2 = \frac{16}{3} - \frac{24}{3} = -\frac{8}{3}$

$\oint_L A \cdot dl$



for l_1 $\quad \vec{dl} = dy\,\hat{a}_y$

$\int_{l_1} \vec{A} \cdot \vec{dl} = \int_0^2 2yz = 0$

for l_2 $\quad \vec{dl} = dy\,\hat{a}_y + dz\,\hat{a}_z$

$\int_{l_2} \vec{A} \cdot \vec{dl} = \int 2yz\, dy + \int \overset{x=0}{\cancel{3xz}}\, dz = 2\int_0^2 yz\, dy = 2\int_0^2 y(2-y)\, dy$

$$\int_{l_2} A \cdot dl = 2\left[\frac{2y^2}{2} - \frac{y^3}{3}\right]_0^2 = 2(4) - 2\left(\frac{8}{3}\right) = 8 - \frac{16}{3} = -\frac{8}{3}$$

for l_3 $\vec{dl} = dz\, \hat{a}_z$

$$\int_{l_3} A \cdot dl = \int_0^2 3xz\, dz = 0$$

$$\oint_L A \cdot dl = -\frac{8}{3} = \int (\nabla \times A) ds$$

(7) $\vec{F} = r^2 \cos(\phi)\, \hat{a}_r + r^2 \cos(\phi)\, \hat{a}_r - r^2 \cos(\theta) \sin(\phi)\, \hat{a}_\phi$

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4 \cos(\phi)\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}\left(r^2 \cos(\phi)\sin(\theta)\right) - \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \phi} r^2\cos(\theta)\sin(\phi)$$

$$\nabla \cdot \vec{F} = 4r\cos(\phi) + \frac{r\cos(\phi)\cancel{\cos(\theta)}}{\cancel{\sin(\theta)}} - \frac{r\cos(\theta)\cancel{\cos(\phi)}}{\cancel{\sin(\theta)}} = 4r\cos(\phi)$$

$$\iiint_V \nabla \cdot \vec{F}\, dv = 4\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^R r^3 \sin(\theta)\cos(\phi)\, dr\, d\theta\, d\phi$$

$$= 4 \left.\frac{r^4}{4}\right|_0^R \left.\frac{\sin^2(\theta)}{2}\right|_0^{\frac{\pi}{2}} \left.(\phi)\right|_0^{\frac{\pi}{2}}$$

$$= R^4\left(\frac{1}{2}\right) \cdot \left(\frac{\pi}{2}\right) = \frac{\pi R^4}{4}$$

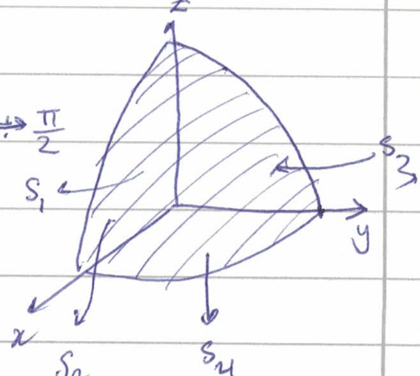
There are 4 surface area.

For S_1 wher $r = R$, $\theta: 0 \to \frac{\pi}{2}$, $\phi: 0 \to \frac{\pi}{2}$

$ds_1 = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$

$$\psi_1 = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} r^4 \sin(\theta)\cos(\theta)\, d\theta\, d\phi$$

$$= \frac{\pi}{2}(R)^4 \left.\frac{\sin^2\theta}{2}\right|_0^{\frac{\pi}{2}} = \frac{\pi R^4}{4}$$



for S_2 :- $\phi = 0$, $r : 0 \to R$, $\theta : 0 \to \frac{\pi}{2}$

$$ds_2 = -r\,dr\,d\theta\,\hat{a}_\phi$$

$$\psi_2 = \int_0^{\frac{\pi}{2}}\int_0^R r^3 \cos(\theta) \underbrace{\cancel{\sin(\phi)}}_{0}\,dr\,d\theta = 0$$

ψ_3 :- $\phi = \frac{\pi}{2}$ $r : 0 \to R$, $\theta : 0 \to \frac{\pi}{2}$

$$ds_3 = r\,dr\,d\theta\,\hat{a}_\phi$$

$$\psi_3 = -\int_0^{\frac{\pi}{2}}\int_0^R r^3 \cos(\theta) \underbrace{\cancel{\sin(\phi)}}_{1}\,dr\,d\theta$$

$$= -\left.\frac{r^4}{4}\right|_0^R \left.\sin(\theta)\right|_0^{\frac{\pi}{2}} = -\frac{R^4}{4}$$

For S_3 :- $\theta = \frac{\pi}{2}$, $\phi : 0 \to \frac{\pi}{2}$, $r : 0 \to R$

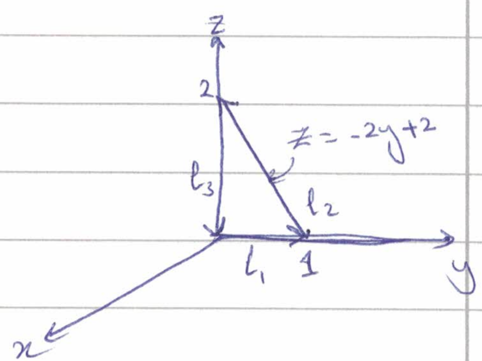
$$ds = r\sin(\theta)\,dr\,d\phi\,\hat{a}_\theta$$

$$\psi_4 = \int_0^{\frac{\pi}{2}}\int_0^R r^3 \cos(\phi) \underbrace{\cancel{\sin(\theta)}}_{1}\,dr\,d\phi$$

$$\psi_4 = \left.\frac{r^4}{4}\right|_0^R \left.\sin(\phi)\right|_0^{\frac{\pi}{2}} = \frac{R^4}{4}$$

$$\psi_{total} = \frac{\pi R^4}{4} + 0 - \frac{R^4}{4} + \frac{R^4}{4} = \frac{\pi R^4}{4} = \oiint \vec{F}\cdot d\vec{s} = \iiint (\nabla\cdot\vec{F})\,dv$$

8) $\vec{A} = 6\hat{a}_x + yz^2\hat{a}_y + (3y+z)\hat{a}_z$



$$\nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6 & yz^2 & 3y+z \end{vmatrix}$$

$$= (3-2zy)\,\hat{a}_x$$

$z = 2-2y$

$dz = -2dy$

$y = \frac{2-z}{2} = 1 - \frac{z}{2}$

$$\iint_S (\nabla\times\vec{A})\,d\vec{s} = \int_0^1\int_0^{2-2y} (3-2zy)\,dz\,dy$$

$$= \int_0^1 \left(3z - z^2 y\right)\Big|_0^{2-2y} dy$$

$$= \int_0^1 \left(6 - 6y - 4y + 8y^2 - 4y^3\right) dy$$

$$= \int_0^1 \left(-4y^3 + 8y^2 - 10y + 6\right) dy = \left(-y^4 + \frac{8y^3}{3} - 5y^2 + 6y\right)\Big|_0^1$$

$$= -1 + \frac{8}{3} - 5 + 6 = -\frac{8}{3}.$$

l_1, $\vec{dl} = dy\,\hat{a}_y$, $z = 0$

$$\int_{l_1} A\,\vec{dl} = \int_0^1 yz^2\,dy = 0$$

For $l_2 \Rightarrow \vec{dl} = dy\,\hat{a}_y + dz\,\hat{a}_z$

$$\int_{l_2} A\cdot dl = \int_1^0 yz^2\,dy + \int_0^2 (3y+z)\,dz$$

$$= \int_1^0 y(2-2y)^2\,dy + \int_0^2 \left(3\left(1-\frac{z}{2}\right) + z\right) dz = \int_1^0 \left(4y - 8y^2 + 4y^3\right) dy + \int_0^2 \left(3 - \frac{z}{2}\right) dz$$

$$= \left(\frac{4y^2}{2} - \frac{8y^3}{3} + y^4\right)\Big|_1^0 + \left(3z - \frac{z^2}{4}\right)\Big|_0^2 = -2 + \frac{8}{3} - 1 + 6 - 1 = \frac{8}{3} + 2 = \frac{14}{3}$$

$\ell_3 \Rightarrow \vec{d\ell} = dz\,\hat{a}_z \qquad y=0$

$$\oint_{\ell_3} A\, d\ell = \int_2^0 (3y+z)\, dz$$

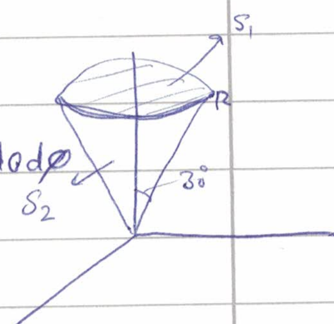
$$= \left(\cancel{3yz} + \frac{z^2}{2}\right)_2^0 = -2$$

$$\oint_L \vec{A}\cdot\vec{d\ell} = \frac{14}{3} - 2 = \frac{8}{3} = \iint_S (\nabla\times\vec{A})\cdot\vec{ds} \qquad \#\#$$

9) $\vec{F} = r^2\sin(\theta)\,\hat{a}_r + 4r^2\cos(\theta)\,\hat{a}_\theta - r^2\tan(\theta)\,\hat{a}_\phi$

$$\nabla\cdot\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4\sin(\theta)\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}\left(\overbrace{4r^2\sin(\theta)\cos(\theta)}^{2r^2\sin 2\theta}\right) - \cancel{\frac{1}{r\sin(\theta)}\frac{\partial}{\partial\phi}\left(r^2\tan(\theta)\right)}$$

$$= 4r\sin(\theta) + \frac{4r\cos(2\theta)}{\sin(\theta)}$$



$$\int_V \nabla\cdot\vec{F}\, dV = \int_0^{2\pi}\int_0^{\pi/6}\int_0^R 4r\left(\sin(\theta) + \frac{\cos(2\theta)}{\sin(\theta)}\right) r^2\sin(\theta)\, dr\, d\theta\, d\phi$$

$$= 4\int_0^{2\pi}\int_0^{\pi/6}\int_0^R r^3\left(\sin^2\theta + \cos(2\theta)\right) dr\, d\theta\, d\phi$$

$$= 4(2\pi)\int_0^R r^3\, dr \int_0^{\pi/6}\left(\sin^2(\theta) + \cos(2\theta)\right) d\theta$$

$\cos(2\theta) = 1 - 2\sin^2(\theta)$

$\sin^2(\theta) = \frac{1 - \cos 2\theta}{2}$

$$= 8\pi\left(\frac{R^4}{4}\right)\int_0^{\pi/6}\left(\frac{1}{2} + \frac{\cos(2\theta)}{2}\right) d\theta = \frac{4\pi R^4}{4}\left(\theta + \frac{\sin(2\theta)}{2}\right)_0^{\pi/6}$$

$$= \frac{\cancel{4}\pi R^4}{\cancel{4}}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right) = \pi R^4\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right) =$$

$$\oint_S F \cdot ds = \Psi_1 + \Psi_1$$

For S_1 :- $r = R$, $\theta : 0 \to \frac{\pi}{6}$, $\phi : 0 \to 2\pi$

$$\vec{ds} = r^2 \sin(\theta)\, d\theta\, d\phi\ \hat{a}_r$$

$$\Psi_1 = \int_0^{2\pi}\int_0^{\frac{\pi}{6}} r^4 \sin^2(\theta)\, d\theta\, d\phi$$

$$= 2\pi R^4 \int_0^{\frac{\pi}{6}} \sin^2(\theta)\, d\theta$$

$$= \frac{2\pi R^4}{2} \int_0^{\frac{\pi}{6}} \left(1 - \cos(2\theta)\right) d\theta$$

$$= \pi R^4 \left[\theta - \frac{\sin(2\theta)}{2}\right]_0^{\frac{\pi}{6}} = \pi R^4 \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right]$$

For S_2 :- $\theta = \frac{\pi}{6}$, $r : 0 \to R$, $\phi : 0 \to 2\pi$

$$ds = r \sin(\theta)\, dr\, d\phi\ \hat{a}_\theta$$

$$\Psi_2 = \int_0^{2\pi}\int_0^{R} 4r^3 \sin(\theta)\cos(\theta)\, d\phi\, dr$$

$$= 4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) 2\pi \left(\frac{r^4}{4}\right)_0^R$$

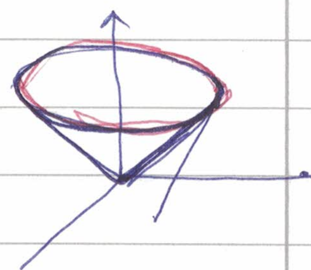
$$= \frac{\sqrt{3}\,\pi R^4}{2}$$

$$\Psi_t = \Psi_1 + \Psi_2 = \pi R^4 \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right] + \frac{\sqrt{3}}{2}\pi R^4$$

$$= \pi R^4 \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{4}\right] = \pi R^4 \left[\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right] = \int_V (\nabla \cdot \vec{F})\, dv \quad \#$$

(10) $\vec{G} = 15\,\hat{a}_r$

$$\nabla \times \vec{G} = \frac{1}{r^2 \sin(\theta)} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & (r\sin\theta)\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 15 & 0 & 0 \end{vmatrix} = 0$$



$$\iint_S (\nabla \times \vec{G})\, ds = 0 \checkmark$$

$$\int G \cdot dl \Rightarrow r=5,\ \theta = 25^\circ,\ 0 \le \phi \le 2\pi$$

$$dl = r\sin\theta\, d\phi\, a_\phi = \cancel{r\sin}$$

$$\int G \cdot dl = \underline{\underline{0}} \checkmark \quad \#$$

(11) $\vec{D} = 2r\,\hat{a}_r\ C/m^2$, find Ψ_{total}

$$Q = \int D \cdot ds = \Psi = \int \rho_v\, dv = \int \nabla \cdot D\, dv$$

$$\nabla \cdot D = \frac{1}{r^2} \frac{\partial}{\partial r} (2r^3) = 6$$

$$\Psi = Q_{enc} = \int_0^{0.4}\int_0^{0.4}\int_0^{0.4} (\nabla \cdot \vec{D})\, dv = 6\, [x]_0^{0.4} [y]_0^{0.4} [z]_0^{0.4}$$

$$= (0.4)^3 (6) = \underline{0.38\,C}$$

(12) $\vec{G} = \dfrac{\cos(\phi)}{\rho}\, \hat{a}_z$

$$\nabla \times \vec{G} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{\cos(\phi)}{\rho} \end{vmatrix} = \frac{-\sin(\phi)}{\rho^2} \hat{a}_\rho + \frac{\cos(\phi)}{\rho^2} \hat{a}_\phi$$

$$ds = \rho \, d\phi \, dz \, \hat{a}_\rho \qquad \rho = 2$$

$$\int_S (\nabla \times \vec{G}) \cdot \vec{ds} = -\frac{1}{\rho} \int_0^2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(\phi) \, d\phi \, dz$$

$$= \frac{-1}{2} \; z \Big|_0^2 \; -\cos(\phi) \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{-1}{2} (2) \cos\left(\frac{\pi}{3}\right) = \frac{-1}{2}$$

$\oint \vec{G} \cdot \vec{dl}$ $\Rightarrow$ Because $\vec{G}$ got components only in z direction

so $\int_{①} G \cdot dl$ & $\int_{③} G \cdot dl = 0$

For l_2 :- $\phi = \frac{\pi}{2}$, $\rho = 2$, $z ; 0 \to 2$

$$\vec{dl} = dz \, \hat{a}_z$$

$$\int_{l_2} \vec{G} \cdot \vec{dl} = \frac{\cos(\phi)}{\rho} \int_0^2 dz = 0$$

For L_4 : $\phi = \frac{\pi}{3}$, $\rho = 2$: $z : 2 \to 0$

$$dl = dz \, \hat{a}_z$$

$$\int_{l_4} \vec{G} \cdot \vec{dl} = \frac{\cos(\phi)}{\rho} \int_2^0 dz = \frac{1}{4} \; z \Big|_2^0 = \frac{-1}{2} = \int_S \nabla \times \vec{G} \cdot \vec{dS} \; \#$$

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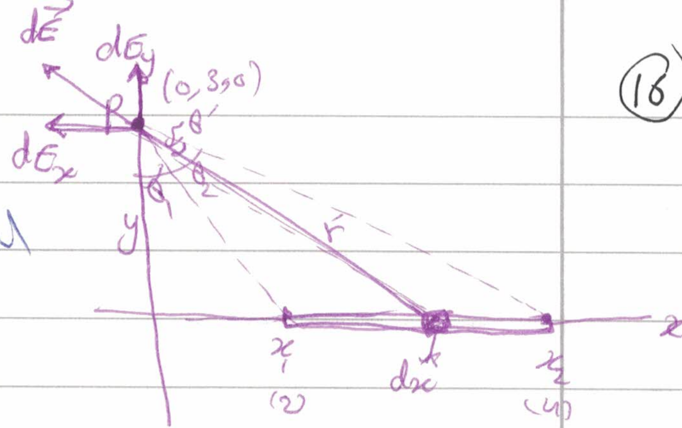


We consider a small differential length $d\vec{x} \Rightarrow dq = \rho_\ell \, dx$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{|r'|^2} \hat{r}$$

because x goes back to zero

$$\hat{r} = \frac{-x}{\sqrt{(3)^2+x^2}} \hat{a}_x + \frac{3}{\sqrt{(3)^2+x^2}} \hat{a}_y$$

$$|r'|^2 = 9+x^2$$

$$d\vec{E} = \frac{\rho_\ell}{4\pi\varepsilon_0} \left[\frac{-x\,dx}{(9+x^2)^{\frac{3}{2}}} \hat{a}_x + \frac{3\,dx}{(9+x^2)^{\frac{3}{2}}} \hat{a}_y \right]$$

$$\vec{E}_x = \frac{-\rho_\ell}{4\pi\varepsilon_0} \int_2^4 \frac{x\,dx}{(9+x^2)^{\frac{3}{2}}}$$

$$\vec{E}_y = \frac{3\rho_\ell}{4\pi\varepsilon_0} \int_2^4 \frac{dx}{(9+x^2)^{\frac{3}{2}}}$$

using:- $$\int \frac{1}{(a^2+x^2)^{\frac{3}{2}}} dx = \frac{x}{a^2\sqrt{a^2+x^2}} + C$$

$$\int \frac{x}{(a^2+x^2)^{\frac{3}{2}}} dx = -\frac{1}{\sqrt{a^2+x^2}} + C$$

$a=3$

So:- $$E_x = \frac{\rho_\ell}{4\pi\varepsilon_0} \left[\frac{1}{\sqrt{9+x^2}} \right]_2^4 = 135 * 10^3 \left[\frac{1}{5} - \frac{1}{\sqrt{13}} \right] = -10.442 \times 10^3 \text{ V/m}$$

$$E_y = \frac{3\rho_\ell}{4\pi\varepsilon_0} \left[\frac{x}{9\sqrt{9+x^2}} \right]_2^4 = \frac{3 * 15 * 9 * 10^3}{9} \left[\frac{4}{5} - \frac{2}{\sqrt{13}} \right]$$

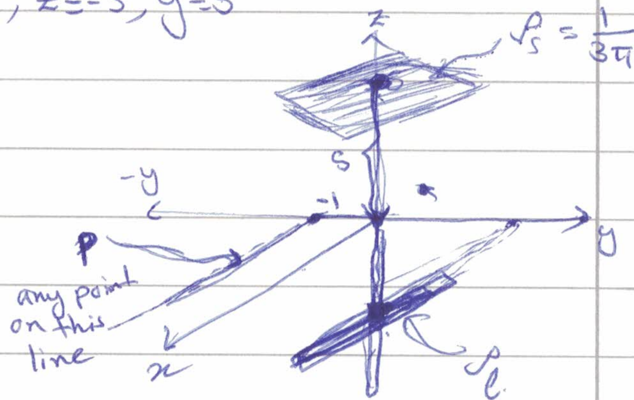
$$= 11.035 * 10^3 \text{ V/m}$$

$$\vec{E} = -10.442 \times 10^3 \, \hat{a}_x + 11.035 * 10^3 \, \hat{a}_y$$

(14)

Determine $\vec{E}$ at $(x,-1,0)$, $\rho_s = \frac{1}{3\pi}\ nC/m^2$ at $z=5$

$\rho_l = \frac{-25}{9}\ nC/m$, $z=-3$, $y=3$



$$\vec{E} = \vec{E}_{line} + \vec{E}_{sheet}$$

$$\vec{E}_{sheet} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a}_n \longrightarrow -\hat{a}_z$$ because the sheet is above observation point

$$\vec{E}_s = \frac{1\times10^{-9}}{6\pi\times8.85\times10^{-12}}\left[-\hat{a}_z\right] = -6\hat{a}_z\ V/m$$

$$\vec{E}_l = \frac{\rho_l}{2\pi\varepsilon_0 r}\,\hat{a}_r$$

We need the normal.

$$\vec{R} = (x,-1,0) - (x,3,-3)$$

$$= (0,-4,3) = -4\hat{a}_y + 3\hat{a}_z$$

$$|R| = 5$$

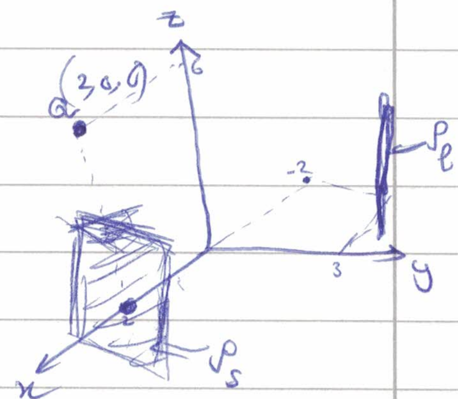
$$\vec{E}_l = \frac{-25\times10^{-9}}{2\pi\times8.85\times10^{-12}\times9\times5}\left[\frac{-4\hat{a}_y+3\hat{a}_z}{5}\right]$$

$$= -2\left[-4\hat{a}_y + 3\hat{a}_z\right] = 8\hat{a}_y - 6\hat{a}_z\ V/m$$

$$\vec{E}_t = 8\hat{a}_y - 12\hat{a}_z\ V/m$$

$Q = 12\,nC$ at $(2,0,6)$, $\rho_\ell = 3\,nC/m$ $x=-2, y=3$

$\rho_s = 0.2\,nC/m^2$ at $x=2$



Find $\vec{E} = \vec{E}_p + \vec{E}_s + E_\ell$

$$\vec{E}_p = \frac{9\times10^9 \times 12\times10^{-9}}{r^2}\,\hat{r}$$

$$\hat{r} = \frac{-2\hat{a}_x - 6\hat{a}_z}{\sqrt{4+36}}$$

$|r| = \sqrt{40}$

$$\vec{E}_p = \frac{9\times12}{(40)^{\frac{3}{2}}}\left(-2\hat{a}_x - 6\hat{a}_z\right)\ V/m = -0.854\,a_x - 2.56\,\hat{a}_z$$

$$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\,a_n \longrightarrow (-a_x)$$

$$= \frac{-0.2\times10^{-9}}{2\times8.85\times10^{-12}}\,a_x = -11.3\,a_x\ V/m$$

$$\vec{E}_\ell = \frac{\rho_\ell}{2\pi\varepsilon_0\,\rho}\,\vec{a}_\rho \qquad \hat{\rho} = (0,0,0)-(-2,3,0) = 2\hat{a}_x - 3\hat{a}_y$$

$|\rho| = \sqrt{13}$

$$\vec{E}_\ell = \frac{3\times10^{-9}}{2\pi\times10^{-12}\times8.85\times\sqrt{13}}\left(\frac{2\hat{a}_x - 3\hat{a}_y}{\sqrt{13}}\right) = 8.3\,\hat{a}_x - 12.45\,\hat{a}_y$$

$$\vec{E}_t = -3.854\,\hat{a}_x - 12.45\,\hat{a}_y - 2.56\,\hat{a}_z\ V/m$$

$*$ $\vec{F} = Q\vec{E} = \left(-38.54\,\hat{a}_x - 124.5\,\hat{a}_y - 25.6\,a_z\right)\mu N$

– The line charge is not in the volume

The sheet will intersect with sphere in circle

$$\psi = Q_{enc} = Q_p + \int \rho_s\,ds = 12\times10^{-9} + 0.2\times10^{-9}\left(\pi\times(2^2)\right)$$

$= 14.5\,nC$

16 $V = \frac{\sin(\theta)}{r^2}$, $\vec{E} = -\nabla V$

$$\vec{E} = -\left(-\frac{2\sin(\theta)}{r^3}\hat{a}_r + \frac{\cos(\theta)}{r^3}\hat{a}_\theta\right)$$

$$\vec{E} = \frac{2\sin(\theta)}{r^3}\hat{a}_r - \frac{\cos(\theta)}{r^3}\hat{a}_\theta$$

$$\rho_v = \nabla\cdot\vec{D} = \varepsilon\,\nabla\cdot\vec{E}$$

$$\nabla\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{2\sin(\theta)}{r}\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}\left(\frac{\sin(\theta)\cos(\theta)}{r^3}\right)$$

$$= \frac{-2}{r^4}\sin(\theta) - \frac{1}{r^4\sin(\theta)}\left(-\sin^2(\theta)+\cos^2(\theta)\right)$$

$$= \cancel{\frac{-2}{r^4}\sin(\theta)} + \cancel{\frac{\sin(\theta)}{r^4}} - \frac{1}{r^4\sin(\theta)} + \cancel{\frac{\sin(\theta)}{r^4}}$$

$$= -\frac{1}{r^4\sin(\theta)}$$

$$\rho_v = \frac{-\varepsilon}{r^4\sin(\theta)} \quad C/m^3$$

17 $\vec{D} = \frac{5}{r^2}\hat{a}_r - r^2\phi\sin(\theta)\,\hat{a}_\phi \quad C/m^2$

$$\rho_v = \nabla\cdot\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(5) - \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}\left(r^2\phi\sin(\theta)\right)$$

$$= \frac{-r}{\cancel{\sin(\theta)}}\left(\cancel{\sin(\theta)}\right) = -r \quad C/m^3$$

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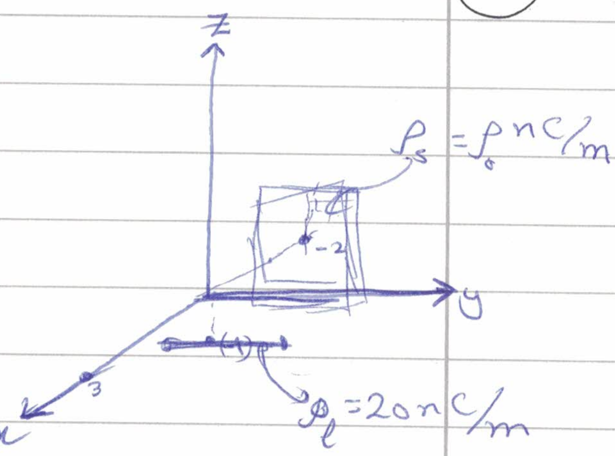
$$E_t = \vec{E}_s + \vec{E}_\ell$$

$$\vec{E}_\ell = \frac{\rho_\ell}{2\pi\varepsilon\rho}\,\hat{a}_\rho$$



$$\rho = (0,0,0) - (3,0,-1) = -3\hat{a}_x + \hat{a}_z$$

$$\vec{E}_\ell = \frac{20\times10^{-9}}{2\pi\times8.85\times10^{-12}\,(10)}\left(-3\hat{a}_x + \hat{a}_z\right)$$

$$= -107.9\,\hat{a}_x + 35.96\,\hat{a}_z \simeq -108\,\hat{a}_x + 36\,\hat{a}_z$$

$$\vec{E}_s = \frac{\rho_s\times10^{-9}}{2\,\varepsilon_0}\,\hat{a}_x = \frac{1\times10^{-9}}{2\times8.85\times10^{-12}}\,\rho_0\,\hat{a}_x = 56.5\,\rho_0\,\hat{a}_x$$

$$\vec{F}_t = -0.6\,\hat{a}_x - 0.18\,\hat{a}_z = Q\vec{E}_t$$

$$E_t = \frac{-0.6\,\hat{a}_x - 0.18\,\hat{a}_z}{-5\times10^{-3}} \quad \cancel{= (-108 + 56.5\rho_0)\,\hat{a}_x}$$

$$= 120\,\hat{a}_x + 36\,\hat{a}_z$$

$$120\,\hat{a}_x = \left(56.5\,\rho_0 - 108\right)\hat{a}_x$$

$$56.5\,\rho_0 = 228 \Rightarrow \rho_0 = 4.0354\ nC/m^2.$$